# A prospect theory-based method for linguistic decision making under risk

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Abstract: Based on the prospect theory, a novel linguistic decision method under risk is proposed. First, the alternatives under each risk state are rated using linguistic terms, and the linguistic decision matrix is constructed. Secondly, the linguistic terms are transformed into triangular fuzzy numbers, so that the linguistic evaluations can be changed into numerical forms. Thirdly, with the aid of the prospect theory, the probability weight functions and the linguistic value functions can be computed, based on which the prospective values of the alternatives are obtained. Finally, the alternatives are ranked with respect to the prospective values combined of probability weight and linguistic value functions. Thus, the optimal choice is made. The decision process takes the psychological preferences of the decision maker into consideration. The practicality of the proposed method is illustrated through an application on stock selection problems.

Key words: decision making under risk; linguistic evaluation; prospect theory; stock selection

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I <sup>n</sup> the real world, decision making problems can be found everywhere. The decision makers must undertake the risk from various possible states. Meanwhile, they must find a proper way to evaluate the alternatives under each state. The problems of decision making under risk have drawn much attention  $[14]$ .

For some decision-making problems, the alternatives are assessed in numerical values <sup>5.9]</sup>. In real decision making, for some attributes, it is difficult to describe them quantitatively. For some other attributes, people prefer to describe them qualitatively, even though they can be described quantitatively. In such cases, the alternatives are available to assess them in linguistic terms instead of in numerical values; i. e. , the decision problems with lin-

guistic evaluation information are common in real life. Clearly, linguistic information is difficult to compute and aggregate directly, so linguistic information must be transformed into numerical information before computation. Some researchers have made contributions to the decision methods on linguistic information  $[10-13]$ . Wang  $[10]$ introduced the extended hesitant fuzzy linguistic term sets and their aggregation in group decision making. Wang et al.<sup>[11]</sup> discussed the multi-criteria group decision-making method based on interval 2-tuple linguistic information and choquet integral aggregation operators. Wang et al.<sup>[12]</sup> provided an uncertain linguistic multi-criteria group decision-making method based on a cloud model. For convenience, in this paper, it is termed linguistic decision making under risk for the problems of decision making under risk with linguistic evaluations.

Moreover, the solution for various problems in economics, as well as in other social sciences, requires understanding of agents' behavior under risk and uncertain $ty^{\lceil 1 \rceil}$ . Therefore, the psychological factors of the decision-makers should be taken into account. Fortunately, the prospect theory is proposed and has been applied into decision-making problems widely  $14-20$ . Yu et al.  $17$  researched the stochastic hybrid multi-attribute decisionmaking method based on the prospect theory. Peng et al.<sup>[19]</sup> discussed the random multi-attribute decision-making methods with trapezoidal fuzzy probability based on the prospect theory. Liu et al.  $[20]$  analyzed risk decision in emergency response based on the cumulative prospect theory. Fan et al. <sup>[21]</sup> discussed multiple attribute decision making ( MADM) with multiple formats of attribute aspirations based on the prospect theory.

Plenty of applications concerning the prospect theory are found  $22-25$ . Wilton et al.  $22$  applied the cumulative prospect theory into reconsidering the capacity credit of wind power. Hansson et al.<sup>[23]</sup> integrated risk-benefit analysis with the prospect theory and discussed decision making for animal health and welfare. Jou et al. [25] provided an application of the cumulative prospect theory to freeway drivers'route choice behaviors.

However, very little research has been conducted concerning the fusion of the prospect theory and linguistic decision making under risk. Thus, in this paper, we provide a method for a linguistic decision making under risk

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based on the prospect theory. Specifically, we primarily introduce a codebook to transform linguistic terms into triangular fuzzy numbers, so that linguistic evaluations can be changed into numerical values. Then, according to the prospect theory, we can compute the probability weight functions and the linguistic value functions, based on which the prospect values of the alternatives are obtained. Finally, the alternatives are ranked in descending order with respect to the prospect values, and the top one is the optimal choice.

#### 1 Decision Methods

In some real decision-making problems, the alternatives are difficult to measure in quantitative forms. In such cases, it is possible to describe the alternatives qualitatively . Therefore, it is important to explore the decision methods with linguistic evaluations. In this paper, we provide some linguistic decision methods under risk.

In the following subsections, we introduce a definition to transform the linguistic terms into fuzzy numbers. Then, for the decision problems with linguistic assessments, linguistic decision methods based on the prospect theory are provided.

## 1. 1 Linguistic decision problems under risk

**Definition 1** The codebook S that can translate each **Definition 1** The codebook *S* that can translate each linguistic term into a triangular fuzzy number  $\tilde{a} = (a^L,$ **Definition 1** The codebook *B* that can translate call<br>nguistic term into a triangular fuzzy number  $\tilde{a} = (a^L,$ <br> $a^W, a^U)$  is defined as shown in Tab. 1. Assume that *S* is a  $a^M$ ,  $a^U$ ) is defined as shown in Tab. 1. Assume that S is a set with seven linguistic terms. Set S can be denoted as S set with seven linguistic terms. Set S can be denoted as S<br>= { $S_{-3}$ ,  $S_{-2}$ ,  $S_{-1}$ ,  $S_0$ ,  $S_1$ ,  $S_2$ ,  $S_3$ } = { high loss, medium loss, low loss, break even, low profit, medium profit, high profit} = { $HL$ ,  $ML$ ,  $LL$ ,  $BE$ ,  $LP$ ,  $MP$ ,  $HP$ }.<br>Then, the terms in set S can be transformed into triangular fuzzy numbers.



The membership function of each term in the codebook  $S$  is illustrated in Fig. 1.

Suppose that  $a = \{a_1, a_2, \ldots, a_m\}$  is the set of the alternatives.  $\boldsymbol{\Theta} = {\theta_1, \theta_2, ..., \theta_n}$  is the set of the states and P





under risk 371<br>= { $p_1, p_2, ..., p_n$ } is the set of the corresponding probabilities. Then, the decision matrix <sup>A</sup> is constructed as follows:

$$
\mathbf{A} = (\tilde{x}_{ij})_{m \times n} = \begin{bmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{bmatrix}
$$
 (1)

 $L x_{m1}$  ...  $x_{mn}$  **J**<br>where  $\tilde{x}_{ii}$  is the value of the alternative  $a_i$  under the state  $\theta$ where and  $\tilde{x}_i$ and  $\tilde{x}_n$  takes the form of the linguistic variables. Let S be and  $\tilde{x}_{ij}$  takes the form of the linguistic variables. Let S be the linguistic term set and  $\tilde{x}_{ij} \in S$ . Let  $\{\theta_1, \theta_2, ..., \theta_n\}$  be a the linguistic term set<br>state vector, where  $p_j$ where  $p_j$  denotes the probability of the state  $\theta_j$ , so that  $\sum_{i=1}^{n}$ , where  $p_j$  denotes the probability of the state<br>  $\sum_{j=1}^{n} p_j = 1$  and  $0 \le p_j \le 1$ ,  $j = 1, 2, ..., n$ . The decision making reference points of different attributes can be expressed as the linguistic variables  $\tilde{x}_i^0$ . Based on  $\int_{i}^{0}$ . Based on these conditions, we can rank the alternatives.

## 1. 2 Combination of the prospect theory with linguistic decision

In real decision making, the psychological attitude / preferences of the decision maker should be taken into ac- $\text{count}^{[26]}$ . Fortunately, the prospect theory developed by Kahneman and Tversky<sup>[27]</sup> is a descriptive model of individual decision making in the cases under risk, which considers psychological factors in decision making. According to the prospect theory, the alternative is measured by the prospective value, which is composed of the probability weight function and the value function.

#### 1. 2. 1 Probability weight function

In this paper, for simplicity, we refer to the result provided by Gonzalez and  $Wu^{[28]}$ . Thus, the probability with respect to each state can be transformed into the weight as follows:

$$
w(p_j) = \frac{p_j^{\gamma}}{(p_j^{\gamma} + (1 - p_j)^{\gamma})^{1/\gamma}}
$$
 (2)

where  $p_i$  is the probability with respect to the state  $\theta_i$ ,  $\gamma$  $= 0.74^{\left[ 28 \right]}$ .

#### 1. 2. 2 Linguistic reference points

The decision reference point represents the equilibrium point of the decision maker's psychological expectations. When the actual result is greater than the decision reference point, the decision maker can obtain happiness, and thus this can be expressed by the gain. On the contrary, when the actual result is less than the decision reference point, the decision maker will feel a sense of frustration, and thus this can be expressed by the  $\cos^{[19]}$ .

In the decision-making problems with linguistic evaluation, the alternatives are all assessed qualitatively instead of quantitatively, and thus the reference point is expressed by the form of linguistic terms. More often, when making a choice, the decision maker has an ideal alternative in mind, which is appropriate to his/her own psychological preference. However, the alternatives of the ideal alternative are difficult to measure by numerical values precisely . For example, when selecting the proper stock, investors might hope to find the optimal one.

Similar to the linguistic assessments of the alternatives, the linguistic reference point can be translated into fuzzy numbers. Therefore, the assessments of the alternatives can be compared to the corresponding linguistic reference point.

#### 1. 2. 3 Linguistic value function

In the prospect theory, an essential feature is that the carriers of value are changes in wealth or welfare, rather than final states. This assumption is compatible with the basic principles of perception and judgment. Human beings' perceptual apparatus is attuned to the evaluation of changes or differences rather than to the evaluation of absolute magnitudes  $27$ .

Meanwhile, the value of a particular change is dependent of initial position. Value should be considered as a function in two arguments: the asset position that serves as a reference point, and the magnitude of the change (positive or negative) from that reference point  $27$ .

Therefore, in accordance with the prospect theory, the outcomes are expressed by means of gains and / or losses from a reference point. The value function in the prospect theory assumes an S-shape concave above the reference point, which reflects the aversion of risk in face of gains; and the convex part below the reference point reflects the propensity to risk in the case of  $losses^{[26]}$ . In fact, the prospect theory has successfully been applied as behavioral models in MADM problems  $^{\lceil 20\text{-}21,24 \rceil}$  .

In the linguistic decision-making problems, before comparison and aggregation, both the linguistic assessments of the alternatives and the reference point should be transformed into numerical values, which take the form of fuzzy numbers in this paper. Then, the gain /loss value functions from the alternatives and the probability weight functions with respect to the states are computed. Ultimately, we can obtain the prospect values, according to which the alternatives are ranked. The decision-making methods are provided in detail in the following subsection.

### 1. 3 Decision methods based on prospect theory

First, the decision maker needs to establish a codebook, which can encode each word in the linguistic term set <sup>S</sup> to <sup>a</sup> triangular fuzzy number. The codebook is shown in Definition 1. It is an advantage that once the codebook has been established, it can be used afterwards.

Since the investor's goal is to make the optimal choice for maximum profits, his/her ratings take the form of linguistic variables.

An individual must fill in a table by completing the following statements: To you, under state  $\theta_i$ , what does the

alternative  $a_i$  seem to be?

The investor should select a proper term to evaluate the The investor should select a proper term to evaluate the alternative  $a_i$  under the state  $\theta_j$  from the linguistic term set  $S = \{S_{-3}, S_{-2}, S_{-1}, S_0, S_1, S_2, S_3\}$ .

"Bull" means that the stock market is a bull market; "Bear" means that the stock market is a bear market; and Steady means a relatively steady stock market.

In order to illustrate what the linguistic ratings might look like, an example is provided in Tab. 2. In the example, the individual's linguistic rating for the alternative  $a_1$ is high profit under the state bull market  $\theta_1$ , break even under the state steady market  $\theta_2$ , and high loss under the state bear market  $\theta_3$ . After the table is filled in, then the linguistic decision matrix <sup>A</sup> is constructed.

	<b>Tab. 2</b> An example of alternatives/statearray		
Alternative $a_i$		θ,	$\theta_3$
a <sub>1</sub>	HР	BE.	HL.
$a_{2}$	MР	LP.	BE.
$a_3$	LP.	BE.	LL.
$a_{\scriptscriptstyle A}$	MР	LL.	MT.

 $\frac{a_4}{a_4}$  MP LL ML ML<br>The decision maker can look up each word  $\tilde{x}_{ii}$  of the linguistic decision matrix  $\boldsymbol{A}$  in the codebook, and transform it into the corresponding triangular fuzzy number. Then, the fuzzy decision matrix can be obtained. For convenience, the fuzzy decision matrix and its triangular fuzzy number are denoted the same as the linguistic ones, fuzzy number are denoted the s<br>i. e.,  $A = (\tilde{x}_{ij})_{m \times n}$  and  $\tilde{x}_{ij} = \{x_{ij}^{\dagger}\}$ ij $\lim_{x \to a} \frac{1}{x}$ ij $e$  as  $\sum_{i=1}^{M} x_{ii}^{\text{U}}$  $\begin{matrix} 0 \\ \vdots \end{matrix}$ .

The decision-making reference points are determined by the DMs'risk preference and psychological state.

The reference points consist of all reference point valwhich are corresponding to each attribute respectively<br>and expressed as linguistic words  $\tilde{x}_i^0$  ( $j = 1, 2, ..., n$ ). Each  $\mathbf{c}$ reference point value can be transformed into a triangular reference point value can<br>fuzzy number, i.e.,  $\tilde{x}_i^0$ an be to<br>an be to<br> $_{i}^{0} = (x_{i}^{10})$ jtransf j $\lim_{x \to 0}$ <br> $\lim_{x \to 0}$  $j^{\text{U0}}$  ).

Based on the fuzzy decision matrix <sup>A</sup> and the reference Based<br>points  $\tilde{x}_i^0$  $\lim_{i \to \infty}$ <br>d on the  $\lim_{i \to \infty} \frac{10}{x_i}$  $\frac{z}{\text{zzy}} \det_{\mathbf{M0}} \mathbf{x}_i^{\text{U0}}$  $j_i^{(0)}$ ), we can construct the gain or loss matrix <sup>R</sup>.

$$
\boldsymbol{R} = (\tilde{r}_{ij})_{m \times n} = (r_{ij}^{\mathrm{L}}, r_{ij}^{\mathrm{M}}, r_{ij}^{\mathrm{U}})_{m \times n}
$$
(3)

 $\boldsymbol{R} = (\tilde{r}_{ij})_{m \times n} = (r_{ij}^L, r_{ij}^m, r_{ij}^v)_{m \times n}$  (3)<br>where the gain/loss  $\tilde{r}_{ij}$  is obtained from the distinction between the evaluation for the alternative and the reference point.

$$
\tilde{r}_{ij} = \tilde{x}_{ij} - \tilde{r}_{j}^{0} = (x_{ij}^{L} - x_{j}^{U0}, x_{ij}^{M} - x_{j}^{M0}, x_{ij}^{U} - x_{j}^{L0})_{m \times n}
$$
 (4)

In accordance with formula  $(2)$ , the value function of the triangular fuzzy number can be calculated as follows:<br>  $\tilde{v}_{ii} = (v_{ii}^{\text{L}}, v_{ii}^{\text{M}}, v_{ii}^{\text{U}}) = (v(r_{ii}^{\text{L}}), v(r_{ii}^{\text{M}}), v(r_{ii}^{\text{U}}))$  (5)

$$
\tilde{v}_{ij} = (v_{ij}^{\mathrm{L}}, v_{ij}^{\mathrm{M}}, v_{ij}^{\mathrm{U}}) = (v(r_{ij}^{\mathrm{L}}), v(r_{ij}^{\mathrm{M}}), v(r_{ij}^{\mathrm{U}}))
$$
 (5)

Then, the value function matrix V can be achieved.<br> $V = (\tilde{v}_{ij})_{m \times n}$ 

$$
\mathbf{V} = \left(\tilde{v}_{ij}\right)_{m \times n} \tag{6}
$$

The probability of each state is provided, and then the probability weight function value of the state  $\theta_i$  can be

computed in accordance with formula  $(2)$ , where  $p_i$  is the probability with respect to the state  $\theta_i$ ,  $\gamma = 0.74^{\lfloor 28 \rfloor}$ .

The probability weights of the states are obtained, and then the prospect value of the alternatives can be computed as follows:

$$
V_i = (V_i^{\text{L}}, V_i^{\text{M}}, V_i^{\text{U}}) = \sum_{j=1}^{n} (w_j \tilde{v}_{ij})
$$
 (7)

The weighted prospect value of the alternatives takes the form of the triangular fuzzy number, so the alternatives are ranked in a descending order based on the defuse seems of the triangular function of the triangular fuzzy number  $V_i$ .<br>fuzzified value  $m(V_i)$  of the triangular fuzzy number  $V_i$ . m

$$
m(V_i) \text{ of the triangular fuzzy number } V_i.
$$

$$
m(V_i) = \frac{V_i^L + 2V_i^M + V_i^U}{4}
$$
(8)

Ultimately, the first one is chosen as the best decision result.

#### 1. 4 Decision-making steps

To make the optimal decision, we provide the operable decision-making steps as follows:

Step 1 Establish the codebook as shown in Tab. 1.

Step 2 Construct the linguistic decision matrix.

Step 3 Transform the linguistic decision matrix to the triangular fuzzy matrix.

Step 4 Select the decision-making reference point.

Step 5 Construct the gain or loss matrix according to formulae  $(3)$  and  $(4)$ .

Step 6 Construct the value function matrix according to formulae  $(5)$  and  $(6)$ .

Step 7 Compute the probability weight value of each state according to formula ( 2) .

Step 8 Compute the prospect value according to formula  $(7)$ .

Step 9 Rank the alternatives according to formula (8) and make the best choice.

In this section, we propose the method on linguistic decision making under risk based on the prospect theory and have provided the operational decision steps. The method integrates the prospect theory into the decision making under risk with linguistic ratings. The feasibility of the proposed method is illustrated with an application on the stock selection in the next section.

#### 2 An Application on the Stock Selection

A stock investor has a moderately large amount of cap-It stock investor that a moderately harge amount of experience in the stock market. Primari-<br>ly, he selects four possible stocks  $\{a_1, a_2, a_3, a_4\}$ . However, he still hesitates on which one is the optimal choice.

Clearly, the stock investor aims to earn the maximum profit. Nevertheless, he must take the risk from the stock market. For simplicity, it is assumed that there are three possible states: the bull market, steady market and bear

market, which can be denoted as the state bull  $\theta_1$ , the state steady  $\theta_2$  and the state bear  $\theta_3$ . Moreover, it is so difficult to predict the selected stock movements even under the assumed market state; i. e. the evaluation for each stock has fuzziness. In such a case, it is available to rate the stock movement in the form of linguistic terms. Therefore, the problem of selecting the most proper stock can be regarded as a linguistic decision-making problem under risk. The proposed method in this paper can be applied.

To make the optimal choice, the investor must predict the probabilities under different states  $\{\theta_1, \theta_2, \theta_3\}$ , and the probabilities under different states { $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ }, and then assess all the possible stocks { $a_1$ ,  $a_2$ ,  $a_3$ ,  $a_4$ } in linthen assess all the possible stocks  $\{a_1, a_2, a_3, a_4\}$  in linguistic terms from set  $S = \{\text{high loss, medium loss, low}\}$ loss, break even, low profit, medium profit, high profit} . Based on the codebook established as Definition 1, the linguistic terms in set  $S$  can be transformed into fuzzy numbers. Then the linguistic rating information can be aggregated so as to find the optimal stock. The detailed decision-making steps are shown as follows.

**Step 1** Construct the linguistic decision matrix

We assume that the investor has rated all the alternatives under each state as shown in Tab. 2, which can also be denoted as Tab. 3.

	<b>Tab. 3</b> Linguistic decision matrix A		
Alternative $a_i$		θ,	$\theta_3$
a <sub>1</sub>	s <sub>3</sub>	$s_0$	$S_{-3}$
a <sub>2</sub>	$s_{2}$	s <sub>1</sub>	$s_0$
$a_3$	s <sub>1</sub>	$s_0$	$S_{-1}$
a <sub>A</sub>	$s_{2}$	$S_{-1}$	$S_{-2}$

Bull means that the stock market is a bull market; bear means that the stock market is a bear market; and steady means a relatively steady stock market.

For example, the investor considers that the stock  $a_2$  is medium profit under the state  $\theta_1$  ( bull market), and the stock  $a_3$  is low loss under the state  $\theta_3$  ( bear market).

Step 2 Transform the linguistic decision matrix into triangular fuzzy matrix EVE THE TREASURE IN EXCREMENT IN THE RESIDENT IN THE RESIDENT INCOMENTATION IN A LOOK up each element  $\tilde{x}_{ii}$  of the linguistic decision ma-

trix in codebook <sup>S</sup>, and transform it into the corresponding triangular fuzzy number. Then the fuzzy decision matrix can be obtained as shown in Tab. 4.

**Step 3** Construct the gain/loss matrix  $R$ 

**Step 3** Construct the gain/loss matrix **R**<br>For simplicity, the reference point is assumed to be  $\tilde{x}_i^0$ For simplicity, the reference point is assumed to be  $\tilde{x}_j^0$ <br>= (0,0,0) ( $j = 1, 2, 3$ ). Thus, the gain/loss matrix is the same as the fuzzy decision matrix, which is also

Tab. 4 The fuzzy gain/loss matrix  $\boldsymbol{R}$ 

<b>Tab. 4</b> The fuzzy gain/loss matrix $\mathbf{R}$			
Alternative $a_i$	$\theta_1$	$\theta_2$	$\theta_2$
$a_1$			$(0.67,1,1)$ $(-0.33,0,0.33)$ $(-1,-1,-0.67)$
a <sub>2</sub>			$(0.33, 0.67, 1)$ $(0, 0.33, 0.67)$ $(-0.33, 0.0.33)$
$a_3$			$(0,0.33,0.67)$ $(-0.33,0.0.33)$ $(-0.67,-0.33,0)$
a <sub>4</sub>			$(0.33, 0.67, 1)$ $(-0.67, -0.33, 0)$ $(-1, -0.67, -0.33)$

shown in Tab. 4.

Step 4 Construct the value function matrix V

The value function matrix  $V$  can be achieved as shown in Tab. 5. Here, we assume that  $\alpha = \beta = 0.88$ , and  $\lambda =$  $2.25^{2}$ . The details are illustrated through an example. 2.  $25^{[2]}$ . The details are illustrated through an example.<br>For instance, the element  $\tilde{r}_{23} = (-0.33, 0, 0.33)$  from For instance, the element  $\tilde{r}_{23} = (-0.33, 0, 0.33)$  from<br>the gain/loss matrix **R** is transformed into the element  $\tilde{v}_{23}$ of the value function matrix <sup>V</sup>.

Tab. 5 The value function matrix V

		<b>Tab. 5</b> The value function matrix $V$	
Alternative $a_i$	$\theta_1$	$\theta$ <sub>2</sub>	$\theta_2$
a <sub>1</sub>			$(0.7,1,1)$ $(-0.85,0,0.38)$ $(-2.25,-1.58,-0.85)$
a <sub>2</sub>			$(0.38, 0.7, 1)$ $(0.0.38, 0.7)$ $(-0.85, 0.0.38)$
a <sub>3</sub>			$(0,0.38,0.7)$ $(-0.85,0,0.38)$ $(-1.58,-0.85,0)$
a <sub>4</sub>			$(0.38, 0.7, 1)$ $(-1.58, -0.85, 0)$ $(-2.25, -1.58, -0.85)$

 $\frac{a_4}{\text{Obviously }r \frac{1}{23}} = -0.33 < 0, \text{ and } r_{23}^M = 0, r_{23}^U = 0.33 > 0, \text{ we have } v_{23}^L = -2.25 \times 0.33^{0.88} = -0.85, v_{23}^M = 0 \text{ and } v_{23}^U = 0.33^{0.88} = 0.38.$  So, the triangular fuzzy number  $\bar{v}_{23}$ .  $v_{23}^{\text{U}}$  = 0.33<sup>0.88</sup> = 0.38. So, the triangular fuzzy number  $\tilde{v}_{23}$  $= (-0.85, 0, 0.38)$ . Similarly, other elements in the value function matrix  $V$  can also be computed.

Step 5 Compute the probability weight value of each state

For simplicity, we assume that the investor has predicted the probabilities of the states in some way. The probabilities under the states bull  $\theta_1$ , steady  $\theta_2$  and bear  $\theta_3$  are  $p_1 = 0.3$ ,  $p_2 = 0.5$  and  $p_3 = 0.2$ , respectively. Then the probability weight function value of the state  $\theta_j$  can be computed in accordance with formula (6), where  $p_j$  is the computed in accordance with formula  $(6)$ , where  $p_i$  is the probability with respect to state  $\theta_j$ ,  $\gamma = 0.74^{\lfloor 28 \rfloor}$ . Specific-<br>ally, the probability weight  $w_j$  ( $j = 1, 2, 3$ ) can be compually, the probability weight  $w_j$  ( $j = 1, 2, 3$ ) can<br>ted as follows:  $w_1 = w(p_1) = \frac{p_1^{\gamma}}{(\gamma^{\gamma})(1-\gamma)}$  $\frac{p_1^{\gamma}}{(p_1^{\gamma} + (1-p_1)^{\gamma})^{1/\gamma}} =$  $0.3^{0.74}$  $\frac{(p_1^{\gamma} + (1 - p_1)^{\gamma})^{1/\gamma}}{(0.3^{0.74} + (1 - 0.3)^{0.74})^{1/0.74}} = 0.33.$  Similarly,  $w_2 =$  $\frac{(0.3^{0.74} + (1 - 0.3)^{0.74})^{1/0.74}}{0.47, w_3 = 0.25$ . It is notable that  $p_1 + p_2 + p_3 = 1$ , but 0.47,  $w_3 = 0.25$ <br>  $w_1 + w_2 + w_3 > 1$ .

 $w_1 + w_2 + w_3 > 1$ .<br>**Step 6** Compute the prospect value  $V_i$ 

Both the probability weights of the states and the value functions have been obtained, then the prospect value of the alternatives can be computed with respect to formula the alternatives can be computed with respect to formula<br>(7). The prospect value  $V_i$  from the alternative  $a_i$  ( $i = 1$ , 2,3,4) is shown in Tab. 6.

<b>Tab. 6</b> The decision result of the alternatives			
Alternatives $a_i$		Prospect value $V_i$ Defuzzified value $m(V_i)$ Rank	
$a_1$	$(-0.73, -0.23, 0.11)$	$-0.27$	
a <sub>2</sub>	$(-0.09, 0.41, 0.75)$	0.37	1
$a_3$	$(-0.79, -0.09, 0.41)$	$-0.14$	$\mathcal{D}_{\mathcal{L}}$
a <sub>4</sub>	$(-1.18, -0.56, 0.12)$	$-0.55$	

The process is illustrated through an element. For the The process is illustrated through an element. For the alternative  $a_3$ , the prospect value  $V_3$  can be computed as<br>  $V_3 = (V_3^L, V_3^M, V_3^U) = \sum_{i=1}^{n} (w_i \tilde{v}_{ii}) =$ 

$$
V_3 = (V_3^L, V_3^M, V_3^U) = \sum_{j=1}^n (w_j \tilde{v}_{ij}) =
$$

$$
0.33 \times (0, 0.38, 0.7) + 0.47 \times
$$
  

$$
(-0.85, 0, 0.35) + 0.25 \times
$$
  

$$
(-1.58, -0.85, 0) =
$$
  

$$
(-0.79, -0.09, 0.41)
$$

Similarly, the prospect values  $V_1$ ,  $V_2$ ,  $V_4$ , can be computed.

Step 7 Rank the alternatives

The weighted prospect value of the alternatives takes the form of the triangular fuzzy number, so the alternatives are ranked in descending order based on the defuzzi-<br>fied value  $m(V_i)$  of the triangular fuzzy number  $\tilde{V}_i$  with  $V_i$  with respect to formula ( 8) .

respect to formula (8).<br>
For example,  $m(V_3) = \frac{V_3^L + 2V_3^M + V_3^U}{4} = [-0.79 + 2 \times$ <br>  $(-0.09) + 0.41]/4 = -0.14$ . Similarly,  $m(V_1)$ ,  $(-0.09) + 0.41$   $1/4 = -0.14$ .<br> $m(V_2)$  and  $m(V_4)$  can be computed.

 $m(V_2)$  and  $m(V_4)$  can be computed.<br>Clearly, stock  $a_2$  should be chosen as the best decision result. Intuitively, stock  $a_2$  will receive medium profit under the state bull  $\theta_1$ , low profit under the state steady  $\theta_2$  and break even under the state bear  $\theta_3$ . In other words,  $a<sub>2</sub>$  is a profitable-to-promise stock almost without the risk  $a_2$  is a profitable-to-promise stock almost without the risk<br>of loss. Thus, stock  $a_2$  is attractive. It is notable that of loss. Thus, stock  $a_2$  is attractive. It is notable that<br>though stock  $a_1$  receives high profit under the state bull though stock  $a_1$  receives high profit under the state bull  $\theta_1$ ,  $a_1$  is not the best for the risk from the high loss under  $\theta_1$ ,  $a_1$  is not the best for the risk from the high loss under<br>the state bear  $\theta_3$ , even though  $a_1$  is no better than stock  $a_3$ with low profit under the state bull  $\theta_1$  and low loss under the state bear  $\theta_3$ .

## 3 Conclusions

We propose a prospect theory-based method for linguistic decision making under risk to solve the decision making under risk problems with linguistic evaluations. The proposed method has the following novelties and characteristics.

1) The alternatives are described qualitatively instead of quantitatively. In some cases, it is so difficult to rate the objects precisely in numerical values. Sometimes, it is even impossible. Thus, it is better for non-professionals to assess some objects by linguistic terms.

2) The probability under different states is transformed into a nonlinear probability weight function. According to the prospect theory, the probability weight function is neither the probability nor a linear function of the probability, but a corresponding weight of the probability.

3) The linguistic evaluations are transformed into linguistic value functions based on the prospect theory. The linguistic value function is expressed by means of gains and/or losses compared to the reference point.

4) The alternatives are ranked with respect to the prospect values combined of probability weight and linguistic value functions. The optimal choice is made. The decision process has taken the psychological preferences from the decision maker into consideration, so that it is much more reasonable.

In future research, we shall continue working on linguistic decision making problems and applications for other domains such as classification, recommendation systems, industrial structure evaluation<sup>[29]</sup> and so on.

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